Unit 5: The Fundamental Theorem of Calculus (FTOC)

Pg. 386-388

FTOC is regarded as one of the most important theorems in all of Calculus.
This theorem serves as the link between Differentiation and Integration as inverse processes.
The statement below says that "if f is integrated and the result is differentiated, we are back at the original function".

Fundamental Theorem of Calculus (FTOC)

-If f is a continuous function on [a,b] and x is a point in (a,b), then $\frac{d}{dx}[\int_a^x f(t)dt] = f(x)$

OR

- $If\ \ g(x) = \int_a^x f(t)dt,\ \ then\ \ g'(x) = f(x)$

Why does this work? (Justify your answer!)

Examples:

1. Find $G'(x)$, if $G(x) = \int_1^x (t^3 - 2t)dt$, then $G'(x) = x^3 - 2x$

2. $G(x) = \int_x^3 (\sin 2t)dt$

3. $G(x) = \int_0^{x^3} (t^2 + 5t)dt$

4. $G(x) = \int_2^{\sin x} (t^2 + t)dt$

5. $G(x) = \int_x^{x^3} \sqrt{1 + t^4}\,dt$

Unit 5: Total Change Theorem

We know that $F'(x)$ represents the rate of change of y = F(x) with respect to x, and [F(b) - F(a)] represents the change in y as x changes from a to b, so this can be constructed to form what we will call the **Total Change Theorem**.

$$\boxed{\int_a^b F'(x)dx = F(b) - F(a)}$$

NOTE: This principle can commonly be applied to rates of change in the natural and social sciences.

*Some examples of the definite integral representing total change***:**

- If V(t) is the volume of water in a reservoir at time *t*, then its derivative $V'(t)$ gives the rate at which water flows into the reservoir at time t.

So, $\int_{t_1}^{t_2} V'(t)dt = V(t_2) - V(t_1)$ **calculates to total change in Volume from t_1 to t_2.**

- If the rate of growth of a population is represented by $\frac{dn}{dt}$, then

So, $\int_{t_1}^{t_2} \frac{dn}{dt}dt = n(t_2) - n(t_1)$ **calculates the increase in population during the time period from t_1 to t_2.**

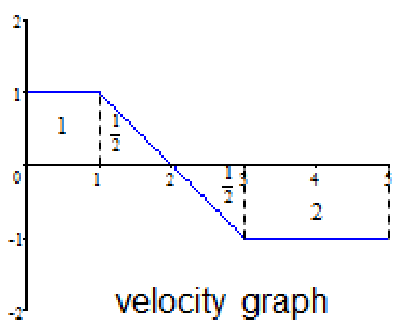
Using the Total Change Theorem to calculate [Displacement vs. Distance]:

To find the displacement (position shift) from the velocity function, we just integrate the function. The negative areas below the x-axis subtract from the total displacement.

$$\text{Displacement} = \int_a^b V(t)\,dt$$

To find distance traveled we have to use absolute value. This will require us to find the roots of v(t) and integrate in pieces.

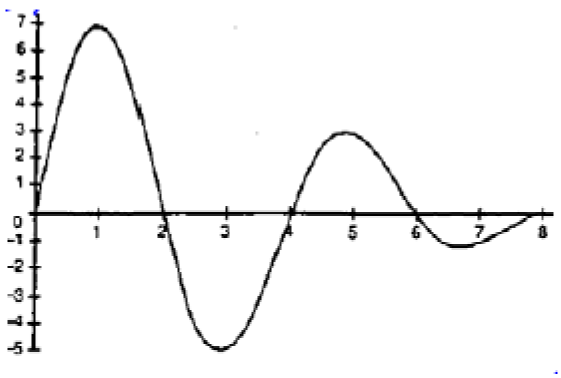
$$\text{Distance Traveled} = \int_a^b |V(t)|\,dt$$



Interpreting Displacement vs. Distance from a graph:

Displacement = ______ Distance = _____

Examples:



Find the times at which:

a. the particle changes direction

b. the particle's acceleration is 0

c. the intervals where the particle is moving in a positive direction

d. the intervals where the particle is moving in a negative direction

e. If the particle begins at a position that is positive, will it ever be located at a position that is negative? Justify your answer.

A particle moves according to: $v(t) = 49 - 9.8t,\ 0 \le t \le 10$

a. Determine when the particle is moving right, left and is stopped.

b. Find the particle's displacement over the given time interval.

c. If the initial position of the particle is at 4, find the particle's position at t=6.

d. Find the total distance travelled by the particle over the interval [0,10].

Unit 5 Worksheet 6 **Name**________________________________

AP Calculus AB **Date** __________________________

In the following problems find $G'(x)$.

1. $G(x) = \int_{-6}^{x} (2t+1)\,dt$

2. $G(x) = \int_{0}^{x} (t^2 + t)\,dt$

3. $G(x) = \int_{1}^{x} \sqrt{1+t^4}\,dt$

4. $G(x) = \int_{0}^{x} \sin^4 t \tan t\,dt$ $-\frac{\pi}{2} < x < \frac{\pi}{2}$

5. $G(x) = \int_{x}^{\pi/4} t \tan t\,dt$ $-\frac{\pi}{2} < x < \frac{\pi}{2}$

6. $G(x) = \int_{2}^{x^3} (t^2 - 5t)\,dt$

7. $G(x) = \int_{0}^{\sin x} (t^2 + \cos t)\,dt$

8. $G(x) = \int_{1}^{x^2+1} \sqrt{2+\sin t}\,dt$

9. $G(x) = \int_{x}^{x^3} \sqrt{1+t^4}\,dt$

10. $G(x) = \int_{\sin x}^{\cos x} t^2\,dt$